ON

THE RELATION WHICH OUGHT TO SUBSIST BETWEEN THE STRENGTH OF AN ELECTRIC CURRENT AND THE DIAMETER OF CONDUCTORS, TO PREVENT OVERHEATING.

A PAPER

READ AT THE

Society of Telegraph-Engineers and Electricians,

BY

Professor GEORGE FORBES, M.A., F.R.S.E.

MARCH 27th, 1884.

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SEC. 1. INTRODUCTORY.

The heating of conductors by the passage of an electric current is injurious to the insulation if the conductor be insulated, and may lead to risks from fire.

In small installations the heating of conductors is always small, because of this fact—that if contractors were to lay down wires so thin that overheating ensued, then we may be sure that the resistance would be far too great for the capabilities of the dynamo machine.

But in large installations, currents of much greater density being carried, the heating may be very great although the resistance of the circuit is small; and it becomes a matter of the utmost importance to know how the heating depends upon the size of conductors and the current density.

I have searched in vain for experimental facts on a large scale, and in absence of these have undertaken the mathematical solution of the problem, and confirmed my results by a few experiments on small currents, besides such isolated examples of measurements of large currents as were available.

I have been at some trouble to determine carefully the nature of conductors which would be required to carry a current capable of supplying 100,000 lamps—say, 70,000 ampères. It may be said that no such conductor would be required—that electricity will be so carried in a network of conductors that in no part will the current carried be excessive. It may further be said that high tension currents will be used to charge accumulators in series, scattered here and there over a district, and that, consequently, small currents only will be required in the mains. To the latter objection, I say that, to carry out some of the provisional orders granted by the Board of Trade, the system of secondary batteries being inadmissible, it will be necessary to carry through the mains, current sufficient for all the lamps. To the former objection, I say that the supply of gas gives us a valuable insight into the similar progress which must be made in the supply of electricity. The problems are remarkably similar, and a due attention to this fact will save the pioneers of electricity much useless expenditure of time, money, and thought. But in gas lighting we carry enormous mains for distances of many miles from the place of manufacture. Witness the huge 4-feet pipes laid through this district last year, to carry gas from Wandsworth to the City. There is no network of conductors here: it is found necessary to carry in one main, gas enough to supply hundreds of thousands of gas lamps. Let it be well noticed, also, that it would be possible to force the gas at high pressure through narrow tubes to fill and supply gas-holders spread about in different parts of a district. The analogy to the proposed system of charging accumulators at high tension is perfect, and this leads me to doubt very much whether the system which has not been found advisable with gas is likely to be successful with electricity.

I still maintain that, to supply the electric light on a large scale, we must face the problem of finding out what conductor will carry a current of 70,000 ampères without overheating.

Now, in doing this we are going a step in advance of what has been done before, just as (to cite, as example, a contemporary engineering work), in designing the Forth Bridge, Messrs. Fowler and Baker are extending the principles of bridge-making to magnitudes hitherto unknown. Here the laws of the stability of bridges are known, and, with experience on smaller bridges, combined with laboratory tests of the strength of materials, a sure advance can be made to the larger structure. It has been my endeavour to find out whether, with the facts at our disposal as to the smaller currents carried by smaller conductors, and the laboratory experiments on the nature of our conductors and insulators, we are in a position to propound laws which shall be a guide to us in extending these principles to the construction of a suitable conductor for very large currents, say, of 70,000 ampères.

Sec. 2. Historical Summary.

In 1882 the Fire Risks Committee of this Society discussed the question, and I believe I am right when I state that it was seriously proposed as a rule, to prevent overheating of the wires, that the permissible current should be so many ampères per square inch section. I have often heard this error repeated. I believe it has actually been adopted by the fire insurance companies as a measure of safety, and a precise 1,000 ampères per square inch has been given as the safe current. With regard to the insurance companies, little harm has been done by this, because they have had only small installations to deal with at present, and, as above stated, there is practically no danger from this cause; but it seems surprising that in one breath they should tell contractors that in small installations they must not raise the temperature of their conductors $\frac{1}{10}$ of a degree, and that in large installations they may make their conductors red-hot.

As a matter of fact, in any installations, except very large ones, the safe conductor ensures greater economy than the unsafe one; and Sir William Thomson has done well* in fixing the size of conductors by commercial considerations, when he showed that the interest and depreciation on the cost of conductors should equal the annual loss of horse-power in heating up these conductors.

There is a limit above which this rule does not apply, because the heating becomes so great that the insulation is injured. The first person who, so far as I

* B. A. Reports, 1881, pp. 518 and 526.

know, has taken notice of this, is Mr. Cowling Welsch, in a table published by Messrs. E. & F. Spon. He fixes the limit at 2,700 ampères, but he does not state what he considers to be the limiting temperature which is tolerable, nor does he specify the nature of the insulation. His facts seem to be taken from the tests of Messrs. Clark, Forde, and Taylor (see next page.) Mr. T. Gray has also taken notice of the failure of Sir William Thomson's law for high currents, in a paper contributed to the *Philosophical Magazine* in 1883, and fixes the limiting value at 5,000 ampères.

I shall not take up the question of how Sir William Thomson's rule is to be applied commercially. I have resolved in this communication to confine myself to one point—the strength of current which can be carried through a wire under different conditions without overheating.

I find that two writers have worked at this subject from a mathematical point of view, and each has worked out some concrete examples. One of these is Mr. Day, of King's College, whose useful little book, "Electric Light Arithmetic," should be studied by all learners. The other author is Mr. T. Gray. His remarks on the subject appear in the *Philosophical Magazine*, September, 1883. In discussing the question of bare wires, both of these authors assume that the cooling effect is proportional to the surface, and they make no reference to the variation from this law which I pointed out in 1882 to the British Association, and which has been confirmed by Mr. Preece. Mr. Day deals only with the case of a naked wire, in which he arrives at the theoretical law—$(\text{current})^2 \propto (\text{diameter})^3$—which I published in 1882, but which I also showed at that time to be contradicted by experiments on a small scale. My own view of the matter is, that while, of course, radiation is proportional to surface, convection is not so, but is nearly constant for rectilinear wires of different diameters but the same temperatures, and that with thin wires consequently convection is the most important factor, but for thick wires radiation proportional to surface is the ruling factor; hence the tables which I have computed are correct for large diameters, but with small wires greater currents may be safely carried.

Mr. Gray has also gone partially into the theory of an insulated cable, and arrived at formulæ very similar to my own.

Some experiments were made by Messrs. Clarke, Forde, and Taylor for the Indian and Oriental Electrical Storage and Works Company, in the last year or two, and the results are published in the *Electrician* for April, 1883. The general conclusion arrived at is, that up to 10 ampères it is safe to allow 1 ampère per 10 pounds of copper per mile, either with naked wire or with insulated wires buried in sand, in the hot Indian climate. They furnish the following table:—

CLARKE, FORDE, AND TAYLOR'S TABLE.

B.W.G.	Diam. Mills.	Weight in lbs. per mile.	Ampères.	Lbs. per ampère.
22	28	12·4	2·33	5·32
21	32	16·2	2·84	5·70
20	35	19·5	3·27	6·00
19	42	28·0	4·3	6·5
18	49	38·1	5·4	7·0
17	58	53·3	6·9	7·7
16	65	67·1	8·3	8·1
15	72	82·5	9·6	8·6
14	83	109·5	11·9	9·1
13	95	143·0	14·56	10·0

No information is given as to the temperature which is considered permissible.

In the second supplement of the *Electrician*, published in March, 1883, a table was printed, supposed to give the currents which could be safely worked through different thicknesses of conductor. This table, however, was founded upon the assumption that the safe-working current was proportional to the sectional area, which is now well known to be far from the case. I quote this simply as one example, out of many which has come to my notice, of the same mistake being made by people who ought to be better informed.

There are five primary cases of conductors which must be treated separately—

(1) Overhead naked wires.
(2) Overhead cables.
(3) Subaqueous cables.
(4) Subterranean and embedded cables.

(5) Coils.

I have added the fifth case, of coils, because it is important in the manufacture of dynamos and magnets.

Each of these classes has its own peculiarities. In all of them heat is generated by the current, and this heat must be got rid of. In case (1) it is got rid of solely by radiation and convection; in the others partly by conduction, and in case (4) very largely by absorption. In case (1) the maximum temperature is reached almost immediately: in some of the other cases it may be many hours before the final steady flow of heat sets in.

SEC. 3. BARE COPPER WIRES.

Having convinced myself that the most satisfactory mode of attacking the problem was to treat it in a strict mathematical way, and being well aware that all the requisite data had been obtained by previous experimenters, I determined to work out practicable tables for the use of electricians from these data, including both bare and insulated conductors. The first step was to solve the following problem:—

Problem I.—*To find the law connecting diameter,* D, *of conductor with that strength of current,* C, *which raises its temperature by a fixed amount t° cent. above that of the surrounding air.*

Let R = the resistance in ohms of a cubic centimètre of the substance of the conductor (= its specific resistance).

Let E = the heat radiated per second from a square centimètre surface when the temperature of the surface is 1° cent. above that of the surrounding air.

The radiation from the surface of 1 cm. length of the wire is $\pi\,\mathrm{D}\,t\,\mathrm{E}$, and this must equal the heat generated in 1 cm. length of the substance $= \mathrm{C}^2 \cdot \dfrac{\mathrm{R}}{\pi\left(\dfrac{\mathrm{D}}{2}\right)^2} \times$ (number of units of heat in 1 joule) $= \mathrm{C}^2 \cdot \dfrac{4\mathrm{R}}{\pi\mathrm{D}^2} \times \cdot 24.$*

* Everett's "Units and Physical Constants." Joule's equivalent of a Gramme centigrade heat unit = $4{\cdot}2 \times 10^7$ ergs, and one joule = 10^7 ergs, $\therefore$ ·24 = number of heat units in one joule.

Whence

$$\pi \, D \, t \, E = C^2 \frac{4R \times \cdot 24}{\pi D^2}$$

and

$$C^2 = D^3 t \cdot \frac{\pi^2 E}{R \times 4 \times \cdot 24} \tag{A}$$

This shows that if the heat be lost by radiation, or by any means which is proportional to the surface, then, in order to keep all the wires of different diameters at the same temperature, we must have the cubes of these diameters proportional to the squares of the currents if the change of resistance with temperature be neglected.

Example:—To take, as a special example, the case of copper we know that

R = ·000001642 ohm* at 0° centigrade,

and increases ·38 per cent. per degree centigrade;

E = ·000168 (polished), or ·000300 (blackened).†

To take an example, let C = 10 ampères; let the wire be No. 16 B.W.G. = 0·165 cm.; the rise of temperature comes out

t = 21·2° C., polished,

or

t = 15·0° C., blackened.

The accompanying Table I. has been computed from the formula obtained above:

$$C^2 = D^3 t \cdot \frac{\pi^2 E}{4R \times 0 \cdot 24}$$

* Maxwell's "Electricity and Magnetism," Vol. I., last chapter.

† This is taken from D. McFarlane's experiments (*Proceedings Royal Society, Edinburgh*, 1872, p. 93), in which radiation took place from balls of considerable size, and, consequently, convection played an unimportant part. If the rise in temperature were 100° or more, it would become necessary to take account of McFarlane's second and even third terms, depending on t^2 and t^3.

TABLE I.

Bare Copper Wires.—Current required to increase the temperature of a copper wire t° centigrade above the surrounding air, the copper wire being bright polished or blackened.

Diameter in centimètres and mills. (thousandths of an inch).		CURRENT IN AMPÈRES.									
		t = 1° C.		t = 9° C.		t = 25° C.		t = 49° C.		t = 81° C.	
Cm.	Mills.	Bright.	Black.	Bright.	Black.	Bright.	Black.	Bright.	Black.	Bright.	Black.
·1	40	1·0	1·4	3·0	4·1	4·8	6·6	6·5	8·9	7·9	11·0
·2	80	2·8	3·9	8·3	11·5	13·5	18·7	18·3	25·3	22·4	31·0
·3	120	5·2	7·2	15·3	21·2	24·9	34·4	33·5	46·4	41·2	57·0
·4	160	8·0	11·0	23·6	32·7	38·3	53·0	51·7	71·5	63·4	87·8
·5	200	11·1	15·4	33·0	45·7	53·5	74·1	72·2	99·9	88·6	123
·6	240	14·6	20·3	43·4	60·0	70·3	97·4	94·9	131	116	161
·7	280	18·5	25·6	54·6	75·6	88·7	123	119	165	147	203
·8	310	22·6	31·2	66·7	92·4	108	150	146	202	179	248
·9	350	26·9	37·3	79·6	110	129	179	174	241	214	296
1·0	390	31·5	43·6	93·3	129	151	210	204	283	251	347
2·0	790	89·2	123	264	365	428	593	577	799	709	981
3·0	1180	164	227	485	671	787	1090	1061	1468	1303	1805
4·0	1570	252	349	746	1035	1211	1675	1633	2260	2006	2776
5·0	1970	353	488	1043	1444	1692	2343	2283	3160	2802	3880
6·0	2360	463	642	1371	1898	2225	3080	3000	4154	3685	5100
7·0	2760	584	808	1728	2392	2803	3882	3781	5233	4642	6426
8·0	3150	714	988	2110	2922	3422	4741	4620	6396	5671	7850
9·0	3540	851	1178	2519	3486	4088	5659	5511	7630	6769	9370
10·0	3940	997	1380	2950	4084	4788	6626	6455	8935	7926	10973
34·4	…	…	…	…	…	…	…	…	…	…	70000

C = current (ampères).

D = diameter of wire (centimètres).

t = excess of temperature (centigrade) above air.

E = coefficient of radiation and convection.

R = specific electrical resistance (ohms).

.24 = number of gramme-centigrade heat units in a watt.

Temperature of the air assumed 20° C.

$$R = 0{\cdot}000001642 \left(1 + \frac{{\cdot}38t}{100}\right).$$

E = ·000168 for polished, ·00032 for blackened, copper.

It gives the rise in temperature in bare copper wires with different currents. In computing with this formula, it must be noticed that the value of R, the specific resistance, varies with the temperature. The resistance at 0° C. is 1642, as stated above. At the temperature of the air (which may be taken at 20° C.) it is 1736, and at any temperature which is t° above 20° C. the resistance is $1642\,(1 + {\cdot}0038\,(t + 20))$. This change of resistance produces a change of 15 per cent. in the current which can be carried at the higher temperatures.

The effects of temperature in altering the resistance are continually cropping up in our application of theory to practice, and the following very striking experiment is worth recording:—

I have been informed by Mr. H. Edmunds that he made experiments with wires $\frac{1}{32}$ inch diameter, flattened out to various widths, through which he passed the current from a machine, the E.M.F. being the same in all the experiments. In the form of wire $\frac{1}{32}$ inch diameter, it was heated to a bright colour; when flattened to $\frac{1}{16}$ and $\frac{1}{8}$ inch width, it lost luminosity; and so on until, when used in a strip $\frac{1}{2}$ inch wide, it kept pretty cool, and fairly stopped the engine. Here we see that the resistance of the wire and all the strips was the same at any constant temperature, but the surface for cooling by radiation and convection was greater with the wider strips. This explains why the wider strips were cooler than the narrower ones, and still more than the wire. Lastly, the resistance is trebled at a temperature which makes the metal barely luminous, and is enormously increased at a bright heat. Hence, in the cases where there was bright luminosity, there was high resistance and less current. The wide strip, being the coolest, had most current, and used up most work, and so stopped the engine.

In the above table (as in the others which follow it), the current specified heats the wire to the degree stated only when steadily applied. A much more powerful current might be used for a very short time at intervals, as in signalling for railways.

The only doubt of the accuracy of the table can come from a doubt as to the accuracy of McFarlane's experiments, which were made in Sir William Thomson's laboratory, or in the extension of his results to surfaces of different dimensions. On this matter I have a few remarks to make.

1. The value which McFarlane found for the loss of heat per second per degree difference of temperature between the metal and the enclosure increased from $t = 5°$ C. to $t = 60°$ C. in the ratio 178 : 226 for polished copper. I have used the value 178 in calculating the above table, so that the current which can be carried with the copper in any state of oxidation, or dirt, is certain to lie between the two values given in the table under *bright* and *blackened*.

2. The only experiments with which I can compare Mr. McFarlane's are those by the late Mr. Nichol, published by Professor Tait in the *Proceedings Royal Society, Edinburgh*, 1869-70, p. 207. The following table gives the comparison.

Loss of heat (per sq. cm. per second per degree centigrade difference of temperature) from copper in air at atmospheric pressure in blackened enclosure at constant temperature (8° C. *in Nichol's experiments*), *for various differences of temperature*:—

Polished.			Blackened.		
Difference of temperature.	Loss per sq. cm. per sec. per degree.		Difference of temperature.	Loss per sq. cm. per sec. per degree.	
	McFarlane.	Nichol.		McFarlane.	Nichol.
Degrees.			Degrees.		
10·0	·000176	…	10·0	·000266	…
12·5	…	·000198	12·5	…	·000364
15·0	·000193	…	19·3	…	·000331
15·3	…	·000182	20·0	·000289	…
20·0	·000201	…	30·0	·000306	…
21·6	…	·000175	33·6	…	·000320
30·0	·000212	…	40·0	·000319	…
32·5	…	·000173	42·2	…	·000322
40·0	·000220	…	50·0	·000326	…
42·5	…	·000173	53·2	…	·000328
50·0	·000225	…	60·0	·000328	…
55·8	…	·000177			
60·0	·000226	…			

A comparison of the results of McFarlane and Nichol shows that they agree generally as well as could possibly be expected, so far as the term which depends on the first power of the temperature in the expression

$$\text{loss of heat} = \mathrm{A}\,t + \mathrm{B}\,t^2 + \mathrm{C}\,t^3 + \cdots$$

is concerned, but that in the comparatively unimportant second term McFarlane makes B negative, and Nichol makes it sometimes positive and sometimes negative. The general conclusion is that we can trust safely to the first term, but that we must not push the application to extremely high temperatures.

3. Both the above sets of experiments were made upon masses of metal some centimètres in diameter, and the conclusion seems warrantable that with

such masses my formula is accurate. I state this now, because I have next to show that the law does not extend to small masses where convection plays a more important part than radiation. My impression is that thin wires lose their heat chiefly by convection when free in the air, but larger masses chiefly by radiation.

I worked at the subject experimentally in 1881 and the following years. My results were published in the *British Association Reports*, 1882; *Annales de l'Electricité*, 15th October, 1882; the *Electrician*, 1882, September, and 1883, February.

My first object in those experiments was to test the correctness of the following considerations:—When a current passes through a wire keeping up a constant temperature, the heat developed by the current over a given length is equal to that lost by radiation, convection, and conduction. It seemed right to suppose that at a fixed temperature this cooling varies as the surface, *i.e.*, as the diameter of the wire. The heat generated by the law of Joule varies as $\mathrm{C}^2\mathrm{R}$ or $\dfrac{\mathrm{C}^2}{\mathrm{D}^2}$, where C = the current, R the resistance, and D the diameter of the wire.

Whence

$$\frac{\mathrm{C}^2}{\mathrm{D}^2} = a\mathrm{D} \text{ (}a\text{ being a constant)},$$

and

$$\mathrm{C} = a\mathrm{D}^{\frac{3}{2}}.$$

To verify the exactness of this law, I experimented on several wires of different diameters but the same conductivity. Each wire was thinly coated with beeswax, whose melting point was 58° C., the temperature of the room being 18° C. A current was passed through one of these wires, and resistances were slowly and gradually removed from the circuit, until the current heated the wire so as to melt the wax. The angle of deflection of the tangent galvanometer was then read off, to give the intensity of the current. The same operation was repeated on the other wires, and the following table gives the results obtained:—

D	C	$\frac{C}{D}$	$\frac{C}{D^{\frac{3}{2}}}$	$\frac{C}{D^2}$
Mm.				
0·58	0·984	1·696	2·229	2·924
1·22	2·304	1·888	1·709	1·548
1·58	3·026	1·915	1·523	1·212

If $C \propto D^{\frac{3}{2}}$, the quotient $\frac{C}{D^{\frac{3}{2}}}$ should be constant for all the wires. If, as some have supposed, $C \propto D^2$, the quotient $\frac{C}{D^2}$ should be constant. If, lastly, $\frac{C}{D}$ is more nearly constant, as is seen to be the case, the law is that the current varies more nearly as the diameter.

Within the last few days I have come across some tests which I had made in 1881, on five thicknesses of lead wire, to find the current required to fuse them. I found that this depended upon the length of the specimen. The reason is that the ends of the wire are clamped by cold metal, which absorbs the heat, and so a greater current is carried without fusion with short specimens than with long ones. I give the results for what they are worth.

Fusing Currents for Lead Wires.

Diameter.	Length.	Fusing Current.
Mm.	Mètre.	Ampères.
0·55	0·025	0·78
0·78	0·025	0·937
0·94	0·025	1·125
{ 1·03	0·025	8·2
{ 1·03	0·225	6·0
{ 1·28	0·300	9·5
{ 1·28	0·150	12·37
{ 1·28	0·075	12·75
{ 1·28	0·050	13·5
{ 1·28	0·025	16·87

These measurements were not made by myself, and I cannot vouch for any very great accuracy. One fact which we learn from them is, that in such experiments, with wires about 1 millimètre thick, the length in experiments of this nature should be not less than 30 centimètres, or, generally, the length should be 300 times the diameter. The effect of using short wires is especially shown with the thicker ones, the experiments on which show that a large quantity of heat is carried off by thermal conduction to the massive cooling terminals.

Taking the case of a long wire, let us see how far it gives us reason to believe in the applicability of the formulæ of this memoir to practical cases. A lead wire, 1·28 millimètre diameter and of considerable length, was heated to the temperature of fusion with a current of 9·5 ampères, and one of 1·03 millimètre diameter, with a current of 6·0: let us find the theoretical current required.

By referring to Problem I., we see that the heat generated per second in one centimètre length of the substance $= C^2 \dfrac{R}{\pi\left(\dfrac{D}{2}\right)^2} \times \cdot 24$ where

C = current in ampères,
R = specific resistance in ohms,
D = diameter.

Now R* = 19,850 at 0° C. for lead in C.G.S. units.
= 44,751 at 327° in C.G.S. units,
= ·000044751 in ohms.

The melting temperature of lead being 327° C.†, or, say, 310° C. above the surrounding air,

$$\therefore \text{heat generated} = \mathrm{C}^2 \frac{\cdot 000044751 \times 4}{\pi \mathrm{D}^2} \times \cdot 24 = \cdot 0000570 \frac{\mathrm{C}^2}{\mathrm{D}^2} \times \cdot 24.$$

Referring to McFarlane's experiment, I find that 60° C. excess of temperature gives a loss of heat per square centimètre per second = ·01356 gramme centigrade heat units with polished copper, and that the loss is nearly proportional to the temperature. This would give ·07006 for 310° C.

The surface of one centimètre length of the first wire is

$$\pi \times \cdot 128 = \cdot 402,$$

and of the second it is

$$\pi \times \cdot 103 = \cdot 324;$$

and the loss of heat is in the first wire

$$\cdot 07 \times \cdot 402 = \cdot 02814,$$

in the second

$$\cdot 07 \times \cdot 324 = \cdot 02268,$$

and this must equal the heat generated as given above, viz.—

$$\begin{aligned} &= \cdot 0000570 \times \cdot 24 \, \frac{\mathrm{C}^2}{\mathrm{D}^2} \\ &= \cdot 00001368 \, \frac{\mathrm{C}^2}{\mathrm{D}^2} \\ &= \cdot 000848 \, \mathrm{C}^2 \text{ for the first wire,} \\ \text{and } &= \cdot 001290 \, \mathrm{C}^2 \text{ for the second wire;} \end{aligned}$$

* Jenkin: Cantor Lectures. † Balfour Stewart: "Elementary Treatise on Heat," p. 88.

whence for the first wire

$$C^2 = \frac{\cdot 02814}{\cdot 000848}$$

and for the second

$$C^2 = \frac{\cdot 02268}{\cdot 001290}$$

which gives us 5·8 and 4·2 ampères theoretically in place of 9·5 and 6·0 respectively, as found by experiment. This only shows that McFarlane's constant does not apply to high temperatures, and that the loss of heat is then much greater than in direct proportion to the temperature.

The only extensive experiments on the subject, with which I am acquainted, have been made by Mr. W. H. Preece, and the results are about to be communicated to the Royal Society. He has been kind enough to show me his experimental results, in order that I might be able to bring before you a comparison with my own results.

He measured the current which was just sufficient to melt platinum wires of different sizes, and he also measured the current which is just sufficient to make wires luminous. The results obtained by Mr. Preece confirm my experiments, and show that with small wires the $(\text{current})^2$ is more nearly proportional to the $(\text{diameter})^2$ than to the $(\text{diameter})^3$.

Sec. 4. Aerial and Subaqueous Cables.

We now come to the case of insulated conductors. There are two cases which can be taken together—aerial and subaqueous. The mathematical treatment of these is, however, not quite the same. In the subaqueous cable we may assume that the outside of the insulator remains at the temperature of the water. In an aerial line it sometimes happens that the insulator is so thin that its outside becomes quite hot. The mathematical view of this case is nearly the same as that of a copper conductor covered with lampblack, which case has already been treated.

Problem II.—*A conductor of radius r_1 is surrounded with an insulator to an outer radius r_2. If the ratio $\frac{r_2}{r_1}$ remains constant, it is required to find the way* *in which the current* C *must vary with radius* r_1*, so that the temperature of the wire shall be* t_1 *degrees cent. above that of the outside of the insulator.*

Let R, as before, be the specific electrical resistance of the conductor in ohms, and let K be the thermal conductivity of the insulator.

Let H be the heat which is generated per second by the current in a length of one centimètre of the conductor.

Then H is also the heat which flows per second radially out of the insulator per centimètre of length. Imagine the insulator to be made up of a number of concentric cylinders, and let the radius of one of them be r and the thickness δr, then the surface of one centimètre length of this cylindrical shell is $2\pi r$; and if $-\delta t$ be the difference of temperature, we have, from Fourier's definition of conductivity,

$$\mathrm{H} = -\mathrm{K} \cdot \frac{2\pi r \cdot \delta t}{\delta r}$$

If we integrate this between the limits $r = r_1$ and $r = r_2$, the difference of temperatures at these points being t_1, we find that

$$\log._e \frac{r_2}{r_1} = \frac{2\pi \mathrm{K}}{\mathrm{H}} t_1$$

Now we also know, from Joule's law, that the heat generated in one centimètre length of the conductor is

$$\mathrm{H} = \frac{4\,\mathrm{C}^2\,\mathrm{R}}{\pi\,\mathrm{D}_1^2} \times (\text{number of heat units in one joule} = 0{\cdot}24).$$

$$\therefore\ \log._e \frac{r_2}{r_1} = \frac{\pi^2\,\mathrm{K}\,\mathrm{D}_1^2}{{\cdot}48\,\mathrm{C}^2\,\mathrm{R}} t_1$$

$$\mathrm{C} = \sqrt{\frac{\pi^2\,\mathrm{D}_1^2 \cdot \mathrm{K}\,t_1}{{\cdot}48\,\mathrm{R}\,\log._e \dfrac{\mathrm{D}_2}{\mathrm{D}_1}}} \qquad (1)$$

It appears from this, that when the ratio $\frac{\mathrm{D}_2}{\mathrm{D}_1}$ is constant, the current must vary as the radius of the conductor to produce a constant difference of temperature between the inside and outside of the insulator. But it would be comparatively useless to tabulate the data from this formula, for with aerial cables we must take note of the excess of temperature of the outside of the insulator over the surrounding air. Call this excess t_2. Then, from the method pursued in the investigation for bare wire, E being, as before, the coefficient of radiation and convection, the flow of heat is

$$= \pi \mathrm{D}_2 t_2 \mathrm{E}$$

but it is also

$$= \frac{2 \pi \mathrm{K} t_1}{\log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1}}$$

whence

$$\frac{t_1}{t_2} = \frac{\mathrm{D}_2 \mathrm{E} \cdot \log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1}}{2 \mathrm{K}}$$

putting E = ·0003 (see above) and K = ·0005

$$\frac{t_1}{t_2} = \frac{3}{10} \mathrm{D}_2 \log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1} \tag{2}$$

$$\therefore t = t_1 + t_2 = t_1 \cdot \frac{10 + 3 \mathrm{D}_2 \log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1}}{3 \mathrm{D}_2 \log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1}} \tag{3}$$

and from (1)

$$\mathrm{C} = \sqrt{\left\{ \frac{\pi^2 \mathrm{K} \mathrm{D}_1^2}{\cdot 48 \mathrm{R}} t \times \frac{3 \mathrm{D}_2}{10 + 3 \mathrm{D}_2 \log \cdot_e \frac{\mathrm{D}_2}{\mathrm{D}_1}} \right\}}$$

This formula is one of great interest. From it we can calculate directly the value of the current or the rise in temperature, when the other quantities are fixed; and all the problems in connection with such cables as are discussed in this memoir can be dealt with by the help of the same formula. There is another matter of great practical importance which it enables us to solve. We can compare it with the formula (A) on page 9, for bare copper wire. Call I and I$'$ the currents in bare and insulated wires, which with the same value of D give also the same value of t.

Assume $\mathrm{E} = \cdot 0003$ for insulation, and $\mathrm{E}' = \cdot 0002$ for copper.

$$\frac{\mathrm{I}^2}{\mathrm{I}'^2} = \frac{\mathrm{D}^3\, t \cdot \dfrac{\pi^2\,\mathrm{E}'}{\mathrm{R}\times 4\times\cdot 24}}{\dfrac{\mathrm{D}_1^2\,\mathrm{D}_2\,\pi^2\,\mathrm{K\,E}\cdot t}{2\times\cdot 24\times\mathrm{R}}\cdot\dfrac{1}{2\,\mathrm{K}+\mathrm{E}\cdot\mathrm{D}_2\,\log\cdot_e\dfrac{\mathrm{D}_2}{\mathrm{D}_1}}}$$

$$= \tfrac{2}{3}\times\frac{\mathrm{D}^3}{\mathrm{D}_1^2\,\mathrm{D}_2}\cdot\frac{1}{2\,\mathrm{K}}\cdot\left(2\,\mathrm{K}+\mathrm{D}_2\,\mathrm{E}\,\log\cdot_e\frac{\mathrm{D}_2}{\mathrm{D}_1}\right)$$

$$= \tfrac{2}{3}\cdot\frac{\mathrm{D}^3}{\mathrm{D}_1^2\,\mathrm{D}_2}\cdot\tfrac{1}{2}\cdot\left(2+\left\{\frac{\mathrm{E}}{\mathrm{K}}=\tfrac{3}{5}\right\}\cdot\mathrm{D}_2\,\log\cdot_e\frac{\mathrm{D}_2}{\mathrm{D}_1}\right)$$

and $\mathrm{D} = \mathrm{D}_1$.

Thus we find that I is greater or less than I$'$, according as

$$2\,\mathrm{D}_1\left(2+\tfrac{3}{5}\,\mathrm{D}_2\log\cdot_e\frac{\mathrm{D}_2}{\mathrm{D}_1}\right)\text{ is }\gtrless 6\,\mathrm{D}_2.$$

Take as special cases (1) $\mathrm{D}_2 = 2\mathrm{D}_1$ and (2) $\mathrm{D}_2 = 4\mathrm{D}_1$. Then I is $\gtrless$ I$'$, according as

$$(1)\ 4+\tfrac{6}{5}\mathrm{D}_2\times\cdot 693\text{ is }\gtrless 6\times 2,$$

$$\text{and (2) } 4+\tfrac{6}{5}\mathrm{D}_2\times 1\cdot 386\text{ is }\gtrless 12\times 2,$$

i.e., according as

$$(1)\ \mathrm{D}_2\text{ is }\gtrless\frac{6\times 2\times 5-20}{6\times\cdot 693}\gtrless 9\cdot 6,$$

$$\text{and (2) }\mathrm{D}_2\text{ is }\gtrless\frac{12\times 2\times 5-20}{6\times 1\cdot 386}\gtrless 12\cdot 0,$$

$$\text{or (1) as } D_1 \text{ is } \gtrless 4{\cdot}8 \text{ centimètres,}$$
$$\text{and (2) as } D_1 \text{ is } \gtrless 3{\cdot}0 \text{ centimètres.}$$

If different values of E and K are adopted, these values will vary proportionally to $\frac{K}{E}$, here assumed to be $\frac{3}{5}$.

We have now arrived at a most important result, viz., that an insulated wire carries a greater current without overheating than a bare wire, if the diameter be not very great. Assuming the diameter of the cable to be twice that of the conductor, a greater current can be carried in insulated cables than in bare wires up to 4·8 centimètres diameter of conductor. But if the insulated cable have a diameter four times that of the conductor, this is the case up to 3·0 centimètres diameter of conductor.

When the thickness of insulation is made very great, the limiting size of conductor which favours the insulated wire is shown below:—

$\frac{\text{Diameter of insulator.}}{\text{Diameter of conductor.}}$		Limiting diameter of conductor which favours insulation.
2	…	4·8 cm.
4	…	3·0 „
6	…	2·5 „
8	…	2·2 „
10	…	2·0 „
100	…	1·0 „

I venture to express the conviction that these results must be looked upon as very surprising. It was hardly to be expected that, by surrounding a copper wire with a bad conductor of heat, we could in any case increase the strength of current which it will carry without overheating. Yet such is clearly the case; and the general explanation of it is that by so doing we increase the surface from which radiation and convection take place. When, however, we have to deal with large currents in large conductors, and the thickness of insulating material is increased in the same ratio, the heat finds greater difficulty in penetrating so thick a mass, and the insulation becomes objectionable from its bad heat-conducting properties, so as to lead us to the result that the bare wire carries more current than the insulated one without overheating, when the diameter is great.

It has been supposed by some persons that heat will escape far more freely through insulating materials, owing to their sometimes being diathermanous, and allowing heat to be radiated through them. Now, in opposition to this view, I must say that very few such substances are diathermanous, and very seldom are they sufficiently homogeneous to allow the possibility of direct radiation through their mass.

Before I go on with what I have to say, I must now pause to make a few remarks on the problem which has just been solved.

1. *Permanent State.*—The definition of Fourier, which has been made the basis of the calculations, has reference to the case only when heat has been steadily supplied for some time, so that the gradation of temperatures from the hot interior and the cool exterior has reached what is called the permanent state. The time which is required to attain this state varies with the conditions of the case. With an insulated cable this time increases with the thickness of the insulating material. It may often happen that many hours must elapse before this state is arrived at, *i.e.*, before the calculations of the present part of my paper can be applied. It must be noticed that previous to the establishment of the permanent state the heating effect is less injurious; so that in all cases where there is a considerable amount of insulating material the current may, during the first working hours, be considerably in excess of what has been calculated out here as the working current. The reason of this is, that during this preliminary stage the heat is used up in raising the temperature of the insulating material, which serves to cool the conductor.

2. *Specific Heat.*—This leads me to my second remark about the above calculations, viz., the influence of specific heat of the insulator. It has been explained that, after the permanent state has set in, the heat which is generated in the conductor all passes through the insulator to the external air. But previous to that time the heat generated is partly used up in raising each layer of the insulator up to the temperature which it must have when in permanent state. The quantity of heat used up in this way depends upon the specific heat of the insulator. The specific heat is the number of units of heat required to raise the temperature of one gramme of the material 1° C. This quantity is known for a large number of substances.

A patent has been taken out for resistances of fine wire through which large currents can be made to pass without undue heating, by embedding the wire in cement or plaster of Paris. The cooling effect of the plaster of Paris is dependent upon its specific heat, and is only temporary. After a very long run of a current through such a conductor, the heating may become greater than in air; and, if the temperature be that of red heat, the plaster becomes a good enough conductor to lower the resistance of the combination so as to make it useless.

I have known of cases where much larger currents have been carried through cables than would be possible by the formula: it is probable in these cases that the current was not continued long enough for the permanent distribution of the temperature to be arrived at. Hence the wire carried a larger current without overheating.

3. Let us form some estimate of the work which is required to heat the insulator to its permanent condition. The exact solution of this problem is troublesome, so we must be content with a very general view of the question. If r_1 and r_2 be the radii of the interior and exterior respectively of the insulator, the mass of this material in a centimètre length is $\pi(r_2^2 - r_1^2)$. Its weight is $w\pi(r_2^2 - r_1^2)$ when w is the specific gravity of the insulator. The heat required to raise its temperature 1° C. is $cw\pi(r_2^2 - r_1^2)$ when c is the specific heat of the material. I can find no determination of the specific heat of gutta percha, but, by the analogy of the substances which it most resembles, it is probably about 0·2. We may take this value for the present, remembering that it is desirable to have experiments made upon all the substances used as insulators, so as to know their specific heats. The density of gutta percha is about 1·0. If we take, as an example, the data derived from an experiment in which C = 500, $r_1 = \cdot 625$, $r_2 = 5$, we find

$$\begin{aligned} cw\pi(r_2^2 - r_1^2) &= 0{\cdot}2 \times 1{\cdot}0 \times 3{\cdot}1416 \times 24{\cdot}6 \\ &= 15{\cdot}3 \text{ heat units per centimètre length.} \end{aligned}$$

And if it is raised on an average 25° C. it requires 25 × 15·3 heat units per centimètre length to establish the state of steady flow.

Now let us see what time it required to generate this heat with the current of 500 ampères.

The specific resistance of copper is ·000001624 ohm at 0° C., ∴ the resistance of one centimètre length of the specified cable is $\frac{\cdot 000001624}{\pi(\cdot 625)^2}$, and the heat generated by 500 ampères is equivalent to $\frac{\cdot 000001624}{\pi(\cdot 625)^2}\times 250{,}000$ watts per centimètre, or $\frac{\cdot 24\times\cdot 1624\times 2\cdot 5}{\pi(\cdot 625)^2}$ heat units per second per centimètre = ·07943 heat units per second per centimètre. But the heat required to warm up the insulator to its permanent state is 15·3 heat units per centimètre length per degree centigrade. Hence, supposing that all the heat generated goes to warm up that insulator, and that none passes through it until the permanent state has been attained, it will take $\frac{15\cdot 3\times 25}{\cdot 079}$ seconds, = 1 hour 21 minutes. It is clear, then, that, since during all this time much heat is passing through, it will be many hours before a current of 500 ampères will be able to heat it as much as is implied by the permanent state.

4. It will be readily believed, from what has been said, how necessary it is to know the thermal conductivity of the insulating material employed in cables. Now, it is very important to notice that the thermal conductivity of substances behaves in the same way as the electrical. The late Principal J. D. Forbes showed that the metals lie in the same order for either conductivity, and that iron becomes a worse conductor for heat at higher temperatures, just as it does for electricity. Now, in the winter of 1872-3, I measured the conductivities of a large number of substances* by an extremely accurate method, consisting of freezing water through them. All the values are less than those which have been obtained by other experimenters at higher temperatures, and the low thermal conductivities of non-metallic substances at low temperatures is completely in accordance with their electric conductivities. It appears, then, that for the high temperatures in conductors the thermal conductivity will be higher, and a larger current can be carried than that given by the formulæ and tables of this memoir.

A few comparisons between the results of Herschel and Lebour, Peclet, and myself will show this.

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* *Proceedings Royal Society, Edinburgh*, 1873.

	G. Forbes.	Herschel.	Peclet.
Marble	·00115 to ·00177	·00470 to ·00560	·0048 to ·0097
Slate	·00081 —	·00315 to ·00550	
Vulcanised rubber	·000089 —	·00034 to ·00055	
Vulcanite	·000083	·00037	
Caoutchouc	—	—	·00041
Gutta percha	—	—	·00048

The average temperature of my results is -10° C; that of the others about +40° C. At about +2° C., Stephan found the conductivity of ebonite (vulcanite) 0·00026.

5. Another point to be considered is, that when exposed to the air the temperature of the outer surface of the insulator is higher than that of the air. If the cable be in water this is not the case, unless excessive currents be used. The case of a cable in water is the most easy to calculate, and is also the most advantageous in practice, as a larger current can be thus conveyed. When it is further considered that under these conditions gutta percha is practically indestructible, we see that in very many cases it will be advantageous to utilise water-power to generate electricity, and the river bed to carry the conductor to the place where the electricity is to be used.

It has often been noticed that the insulation of leads is unaffected by a few hours' run, but is quite hot and soft after twenty-four or thirty hours. This is completely accounted for by what has now been said.

A general result of this investigation is that an electric insulator should have as high a thermal conductivity and as high a specific heat as possible.

Having now discussed fully the conditions of the problem of a cable in air or water, I have computed a table for wires from 1 mm. to 10 cm. diameter, in which the diameter of the insulated cable is four times that of the conductor (this being, as I find from makers' price lists, a common ratio), showing the current which will raise the temperatures t° C. above those of the surrounding air. [This table is substituted for one exhibited to the Society, as it is of more practical value.]

TABLE II.

Subaqueous and Aërial Cables (insulated)—

$$\frac{\text{Diameter of cable}}{\text{Diameter of conductor}} = 4.$$

Temperature of air = 20° C.
t = excess of temperature of conductor over air.

diameter in centimètres and mills.		CURRENT IN AMPÈRES.				
		t = 1° C.	t = 9° C.	t = 25° C.	t = 49° C.	t = 81° C.
Cm.	Mills.					
·1	40	3·7	11·0	17·8	24·0	29·5
·2	80	9·1	27·0	43·8	59·0	72·5
·3	120	15·0	44·4	72·1	97·3	119
·4	160	21·2	62·5	102	137	168
·5	200	27·4	81·0	131	177	218
·6	240	33·7	100	164	219	268
·7	280	40·1	119	192	259	319
·8	310	46·4	137	223	301	369
·9	350	52·9	157	253	342	420
1·0	390	59·3	175	285	384	472
2·0	780	124	367	595	803	988
3·0	1180	189	559	908	1225	1503
4·0	1570	254	753	1221	1646	2021
5·0	1970	319	945	1534	2068	2523
6·0	2360	385	1138	1846	2491	3058
7·0	2760	450	1330	2158	2846	3575
8·0	3150	514	1525	2472	3335	4094
9·0	3540	580	1716	2785	3755	4611
10·0	3940	645	1909	3097	4178	5130

Computed from the formula,

$$C = \sqrt{\left\{\frac{\pi^2 D_1^2 K}{\cdot 48\, R} . t \times \frac{3\, D_2}{10 + 3\, D_2 \log ._e \dfrac{D_2}{D_1}}\right\}}$$

K = thermal conductivity of insulator, = ·00048 for gutta percha;
E = coefficient of cooling, = ·0003.

If, as is possible, the thermal conductivity of an insulating covering were only 0·0003, then a change of K to this amount can be approximately allowed for by multiplying the currents in the table by a factor which varies from 0·95 to 0·84 for the first ten (incl.), and from 0·84 to 0·78 for the last ten sizes (incl.) of conductors.

Sec. 5. Buried Conductors.

The case of conductors buried underground is very difficult to treat mathematically. At present I content myself with a study of a specially favourable case, viz., when the conductor takes the form of a thin sheet lying in a horizontal plane under the ground. This form requires far less metal than any other. The heat which is created by the current is at first largely absorbed in heating up the earth near to it, but after some hours a tolerably permanent state sets in, when a very small amount of heat is still penetrating downwards; but the greater part is conducted through the superincumbent earth and paving, and thence cooled by radiation and convection.

With regard to conduction into the soil, we have some experience from the observations which have been made on the temperature at various depths in the soil or in rock. The daily and yearly variations of temperature produce waves of heat in the soil, which are slowly propagated. At a depth of 25 feet the maximum heat occurs in midwinter, and the annual variation of temperature is only $\frac{1}{23}$ of what it is at the surface.

At a depth of two feet the daily variations of temperature are barely perceptible. In the buried cable the heat generated in the dark hours will not all be dissipated at once, but there will be a steady flow of heat at the surface day and night.

Having stated these preliminary facts, I shall now attempt an approximate solution of our problem.

Problem III.—*A sheet of copper* 1 *centimètre thick and of a width b, buried at a depth d, carries a current C with a rise of temperature t above the surface of the ground, the ground being t′° above the surrounding air. When the steady flow of heat has set in, find the relation between these quantities.*

The heat generated per centimètre length per second

$$= \frac{\mathrm{C}^2\,\mathrm{R} \times {\cdot}24}{b};$$

The heat radiated*

$$= {\cdot}0003 \times b \times t';$$

and these are equal.

$$\therefore \mathrm{C}^2 = \frac{b^2\,t'}{800\,\mathrm{R}}.$$

It would not be permissible to have the surface of the ground raised more than 5° C. in summer. Let us take 10° as a maximum value for t', $\therefore \mathrm{C}^2 = \dfrac{b^2}{80\,\mathrm{R}}$.

We have also the equation of conductivity (N.B.—Conductivity of paving stones and similar materials is ·004 to ·005, according to the experiments and deductions of Peclet, Herschel and Lebour, J. D. Forbes, Thomson and Everett, Ayrton and Perry).

$$\frac{\mathrm{C}^2\,\mathrm{R} \times {\cdot}24}{b} = \text{heat conducted} = \frac{\mathrm{K}\,b\,t}{d} = {\cdot}004 \times \frac{b\,t}{d}$$

$$\therefore \mathrm{C}^2 = \frac{{\cdot}004 \times b^2\,t}{{\cdot}24\,\mathrm{R} \cdot d} = \frac{b^2}{80\,\mathrm{R}}$$

$$\therefore \frac{{\cdot}004\,t}{{\cdot}24\,d} = \frac{1}{80} \qquad t = {\cdot}75 \times d.$$

* It is assumed that the coefficient of cooling for the ground is the same as for a ball freely suspended. It is probably less.

With a depth of 2 feet, or 60 centimètres, the rise of temperature from the surface of the earth to the conductor is 45°, and the difference of temperature between the conductor and surrounding air is 55° C. We might place the conductor under the foot pavement or street close to the surface. This would diminish the value of t, but practically we are not injured by taking a depth of 2 feet, as 50° C. is a permissible rise of temperature. Up to this depth, then, we need only consider the rate at which we can get rid of heat by cooling. We have the equation

$$C^2 = \frac{b^2}{80\,R}$$

and for 50° above the temperature of the air, assumed to be 15°, we have

$$R = 2{\cdot}031 \times 10^{-6}\,\text{ohm},$$

$$\therefore C = \frac{b \times 10^3}{\sqrt{162{\cdot}48}} = 25\,b.$$

The following table is calculated on these principles:—

TABLE III.

Underground flat conductor of copper 1 cm. thick at a depth less than 2 feet below the surface, raising temperature of conductor less than 50°C., and surface of ground 10°C. above air.

Width of copper conductor 1 cm. thick.	Equivalent diameter of same section.	Current to raise surface temperature 10° C.
Cm.	Cm.	Ampères.
10	3·5	250
40	7·0	1,000
90	10·5	2,250
160	14·0	4,000
360	21·0	9,000
2,800	67·5	70,000

The second column is given to show the diameter of wire which would give the same quantity of metal, merely to admit of a comparison with the aerial conductors.

It appears, then, that if $c = 70{,}000$ ampères, we must have $b = 2{,}800$ centimètres, or the strip of copper must be 28 mètres wide. This gives the least weight of copper permissible (unless, indeed, we were to diminish still further the thickness), and it gives us a section = 53 centimètres square of pure copper, equivalent to a diameter of 67·5 centimètres.

I wish here to insist very positively upon the necessity of making such conductors for large currents in the form of flat sheets. Further, when we see the enormous mass of metal required, it is clearly necessary to use iron, which reduces the cost of material to about $\frac{1}{6}$. I hold this view very strongly in opposition to my friend Mr. Preece, who maintains that for electric light leads very pure copper should be used.

SEC. 6. COILS.

There is a very important application of the principles which have here been enunciated, and this is to the currents which can be carried by coils of wire when the wire is varied in thickness or the coil is varied in size. I have been sometimes simply astounded to see the waste of labour, time, and money which have been lavished on determining the proper thickness of wire with which to wind a coil without overheating, for a dynamo field-magnet, for regulating coils in arc lamps, etc., while the whole thing could be done by calculation. My first experiments to test the truth of the theory were undertaken two years ago. I first attacked the question as follows:—Having two coils of the same size, but rolled with wires of different diameters, so as to occupy the same volume and to have the same weight, then if the number of turns on each bobbin is considerable, the length of wire is proportional to $\frac{1}{\mathrm{D}^2}$ (D being the diameter of the wire), and the cross-section is proportional to D^2, hence the resistance of each coil is proportional to $\frac{1}{\mathrm{D}^4}$, and the heat developed in the circuit during a unit of time varies as $\mathrm{C}^2\,\mathrm{R}$ or $\frac{\mathrm{C}^2}{\mathrm{D}^4}$. But the radiating surface of the two coils is the same; whence, if the temperature be the same in both, the heat lost by radiation and convection in a given time is constant, and the heat created in unit time must be constant.

Whence

$$\frac{\mathrm{C}^2}{\mathrm{D}^4} = a^2, \text{ where } a \text{ is constant,}$$

and

$$\mathrm{C} = a\,\mathrm{D}^2;$$

i.e., to attain the same temperature with equal-sized bobbins, the current varies as the section of the wire.

To test the truth of this law, I prepared two copper tubes with flanges at each extremity, and closed at one end, and wound equal weights of two kinds of wire of different diameters. Filling one with water, and inserting a thermometer, the current was increased very slowly until a fixed temperature, sufficiently high, was steadily maintained. The tangent galvanometer was then read off. Afterwards the same temperature was attained with the other coil in the same way. The law above stated was exactly confirmed: the tangents of the angles were proportional to the squares of the diameters.

I next tested the theory in the case of the following problem:—*Two coils of similar shape have their linear dimensions in the ratio* $n : 1$, *and the thickness of wire in the same ratio: what ratio of currents is required to raise both to the same temperature?*

Let C R H be the current, resistance, and heat generated in the smaller coil, and C′ R′ H′ the corresponding quantities in the larger coil.

Then we have

$$\mathrm{C}^2\,\mathrm{R} = a\,\mathrm{H}, \text{ when } a \text{ is a constant.}$$

$$\mathrm{C}'^2\,\mathrm{R}' = a\,\mathrm{H}'$$

$$\mathrm{R}' = \frac{\mathrm{R}}{n}$$

Also, since the temperatures are equal, the heat radiated and convected in the two cases is in proportion to the surfaces, or as $n^2 : 1$, and this is equal to the heat generated, wherefore

$$H' = n^2 H,$$

or

$$C'^2 R' = n^2 C^2 R,$$

or

$$C'^2 \frac{R}{n} = n^2 C^2 R,$$

and

$$C'^2 = n^3 C^2;$$

i.e., the squares of the currents are proportional to the cubes of the linear dimensions. To test this, Mr. R. Goodwin made for me the following experiments:—Coils of wire were wound upon two bobbins whose dimensions are—

Small coil—Length = 50 mm.;
Diameter of tube = 13 mm.;
Diameter of wire = 1·05 mm.
Large coil—Length = 100 mm.;
Diameter of tube = 25·7 mm.
Diameter of wire = 2·09 mm.

Here $n = 2$ and $n^{\frac{3}{2}} = 2{\cdot}83 = \frac{C'}{C}$ theoretically.

At the temperature of 63° C., the tangent galvanometer showed a deflection of 37° with the small coil, and at the same temperature with the large coil the deflection of 64°. The tangents of these angles are 0·75355 and 2·0503 respectively, the ratio of which is 2·72 : 1, which agrees well with the theoretical value.

It becomes an easy thing in any case to calculate the heating of a coil when we know its cooling surface and its resistance.

Let ρ = the resistance of a coil in ohms at the permissible temperature,
S = the surface exposed to the air measured in centimètres,
t = the rise in temperature,
C = the current in ampères.

$$\cdot 24\,\mathrm{C}^2\,\rho = \text{heat generated} = \mathrm{E}\,t\,\mathrm{S},$$

where E is McFarlane's constant varying from ·0002 to ·0003. The latter value may be taken. If 50° C. be the permissible rise in temperature

$$\mathrm{C} = \sqrt{\frac{\cdot 0003 \times 50 \times \mathrm{S}}{\cdot 24 \times \rho}} = \cdot 25\sqrt{\frac{\mathrm{S}}{\rho}}$$

N.B.—It must be remembered in practice that the resistance (cold) must be increased by $\frac{1}{5}$ of its value to give ρ.

Example:—The resistance of the field-magnets of a dynamo is 1·5 ohms cold, and the surface exposed to the air is 1 metre: find the current to heat it not more than 50° C. Here S = 10,000, $\rho = 1{\cdot}8$ ohms, and $\mathrm{C} = \cdot 25\sqrt{\frac{10{,}000}{1{\cdot}8}} = 33{\cdot}5$ ampères. Those who are accustomed to handling dynamos will know that this is very much what we actually find, and it gives us confidence in the applications of theory.

Sec. 7. Conclusions.

The general result of this research seems to be that we have the factors required for arriving at correct theoretical results. The general accordance of the theory with the few experimental facts which I have been able to get hold of, give a confidence in the application of these principles.

1. One of the most important results I have obtained is that the insulation of an aerial conductor is favourable, and gives us a power of using larger currents with conductors of the same size, when the diameters are not very great.

2. In small installations, the question of safety from fire or injury to insulation is not likely to crop up, but the tables here calculated will always be useful to let us know the amount of heating.

3. It is satisfactory to have a set of tables which will give us the increase of temperature with any arrangement of currents and size of wires. But these tables which I have worked out are not (except in the case of large currents) a measure of the best size of conductor for any special installation. They indicate only the safety from heating.

4. In buried conductors the mass of metal must be very great indeed, the inferior limit being set by the amount of heating of the surface of the ground which is permissible.

5. In all ordinary applications of coils of wire we can calculate the rise of temperature, experiment and theory being found to be in accordance.

Transcriber's notes

Obvious typographical errors have been corrected.

Inconsistent spelling has been retained.

The references in the text to Table I. and the following table have been altered slightly to allow for the tables being moved to new pages.

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